Quantum Geometry and the Fermilab Holometer

Chris Stoughton

Fermi National Accelerator Laboratory
Center for Particle Astrophysics

FNAL: A. Chou, H. Glass, C. Hogan, C. Stoughton, R. Tomlin, J. Volk
University of Chicago: B. Lanza, L. McCuller, S. Meyer, J. Richardson
MIT: M. Evans, S. Waldman, R. Weiss
University of Michigan: D. Gustafson
Northwestern: J. Steffen
Vanderbilt: B. Kamai

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Abstract: The Fermilab Holometer

Main Points

- **Hypothesis:** $[X_i, X_j] = i\ell_p\epsilon_{ijk}X_k$
- $\langle x^2_\perp \rangle = L\ell_p'$
- **We are building interferometers to search for macroscopic effects**

- [http://holometer.fnal.gov](http://holometer.fnal.gov) especially scientific bibliography
- Craig Hogan’s papers on arXiv re quantum geometry and interferometers
Context: Vacuum Vessels for Interferometers
Context: Vacuum Vessels for End Mirror
Context: The Neighborhood
2010 x 1113 ft rectangular interferometer in Clearing, IL (1925)
12-inch steel water pipes, evacuated to 13 Torr. Mirrors adjusted by mechanical feedthroughs, coordinated via telephone.
Motivation: Natural Units

Max Planck: from three physical constants, $c, \hbar, G$, derive:

Length: $\ell_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35}\text{m}$

Time: $t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44}\text{s}$

Mass: $m_P = \sqrt{\frac{\hbar c}{G}} = 2.2 \times 10^{-8}\text{kg}$
Motivation: Size and Energy
Motivation: Theoretical Developments

Thermodynamics

- \( S_{BH} = 4\pi (R/2\ell_P)^2 \)
- Formulations of GR from thermodynamics

Holographic Principle

- “Nature’s book keeping system: the data can be written onto a surface, and the pen with which the data are written has a finite size.” - Gerard ’t Hooft
- Verlinde\(^1\) calculates the number of states in a sphere:
  \[ N_G(R) = 4\pi (R/\ell_P)^2. \]

Treat space-time as a statistical behavior of a quantum system. States have new forms of spatially nonlocal entanglement.

\(^1\)JHEP 1104, 029 (2011)
Motivation: Geometry

Paradox

- Classical geometry is based on the primitive notion of spatial points; locality is presumed
- Quantum Physics does not ascribe events to definite points
- How does geometry work for quantum measurements?

Standard Architecture of Physics

- Dynamic space-time responds to particles and fields (lensing; pulsar spin-down) but is not quantum
- Quantum particles and fields defined on the “stage” of classical geometry
- This accommodation explains all quantum mechanics (particle) experiments

It cannot be the whole story
Hypothesis: a new commutator

\[ [x_\mu, x_\nu] = i x^\kappa U^\lambda \epsilon_{\mu \nu \kappa \lambda} \ell'_P \]

- \( x \) are Hermitian operators on bodies
- eigenvalues of \( x \) are locations of bodies
- \( U \) is 4-velocity: \( \partial x / c \partial t \)
- \( \epsilon_{\mu \nu \kappa \lambda} \) is antisymmetric 4-Tensor
- the form is covariant
- describes quantum relationship between two timelike trajectories
Hypothesis: Macroscopic Uncertainty

\[ [x_i, x_j] = i x_k \varepsilon_{ijk} \ell'_P \]

In the Rest Frame

- In the rest frame, turns into Planck scale quantum algebra for position operators in 3D at one time
- Similar to angular momentum algebra, with x in place of J
- Uncertainty in transverse position after propagating D:
  \[ \langle x^2_\perp \rangle = D \ell'_P \]
- Uncertainty increases with separation D.
- This is a “new” quantum departure from classical geometry
- Purely transverse to propagation direction
Hypothesis: Transverse Position Uncertainty
Hypothesis: Normalization

Define the state $|d\rangle$ as two bodies separated by distance $D = d\ell'_{P}$.

For a given separation $D$, there are $2d + 1$ states.

In a sphere, values $d$ are allowed such that $0 \leq d \leq R/\ell'_{P}$. Each one of these has $2d + 1$ states. So the number of states in a 3-sphere of radius $R$ is

$$N_{3S} = \sum_{d=0}^{R/\ell'_{P}} 2d + 1 = d(d + 1) \rightarrow (R/\ell'_{P})^2 \text{ for } R \gg \ell'_{P}$$

- Recall that Verlinde calculates $N_G(R) = 4\pi (R/\ell_{P})^2$
- $N_{3S}(R) = N_G(R) \rightarrow \ell'_{P} = \ell_{P}/2\sqrt{\pi}$
- The normalized transverse position uncertainty is

$$\langle x^2 \rangle = D\ell_{P}/2\sqrt{\pi}$$
Measure voltage at detector as a function of time.

For a constant number photons per second at the input, this measures

\[ \Delta(t) \equiv D_2(t) - D_1(t) \]
Video demonstration of $\Delta(t)$ at audio frequencies.
Reference Design for One Interferometer

- Arm Length $L = 40$ meters
- Optical Wavelength $\lambda = 1.064$ microns
- Power on beam splitter $P_{BS} = 1$ kWatt
- Free Spectral Range $f_c \equiv c/2L = 3.75$ MHz
- Round Trip Time $\tau_c \equiv 2L/c = 267$ nanoseconds
Experiment: Control two degrees of freedom

\[ \Delta \equiv D2 - D1 \]

- End mirrors actuated by three spring-loaded piezo crystals
- 2 kHz resonant frequency
- 14 micron range
- Drive north and east arms differentially to control \( \Delta \)

\( \lambda_{\text{laser}} \)

- Power Recycling Mirror (PRM) reflects photons back towards the beam splitter; makes IFO an optical cavity
- Change \( \lambda_{\text{laser}} \) to minimize reflection of input light from the PRM
Experiment: Parts List

- 2W (continuous) Nd:YAG infrared laser
- Beam splitter substrate: polished flat to lose < 50 ppm to scattering; Suprasil 3001 (loss < 1 ppm)
- End mirrors: 78m radius; Ion Beam Sputtering (loss < 10 ppm)
- Vacuum System: 304L stainless steel + conflats; free of Hydrocarbon contamination
- Control System: Digital filtering w/FPGA (NI PXIe-7852R) to control end mirrors ($\Delta L$ and alignment)
- Cross Correlation: 4 channels @ 100 MHz (NI PXIe-5122) and 32-core computer
Experiment: Seismic Noise Suppression
Autocorrelation function is $R_{\Delta \Delta}(\tau) \equiv E[\Delta(t)\Delta(t + \tau)]$

At zero time lag, it is the variance: $R_{\Delta \Delta}(0) = \langle x^2 \rangle$

Linear extrapolation to zero at $t_c$

Predict $\Xi(f)$ from theoretical $R_{\Delta \Delta}$

Calculate $\Xi(f)$ from measured $\Delta(t)$

Inverse Fourier Transform yields measured $R_{\Delta \Delta}$
Length Spectral Density

- Holographic Signal for $f < f_c$:
  \[
  \sqrt{\Xi^H(f)} = 2L \sqrt{\ell_P/c} \sqrt{\pi} = 1.39 \times 10^{-20} \text{meters}/\sqrt{\text{Hz}}
  \]

- Poisson (shot) Noise:
  \[
  \sqrt{\Xi^{\text{shot}}(f)} = \sqrt{hc\lambda/4\pi^2 P_{BS}} = 1.64 \times 10^{-18} \text{meters}/\sqrt{\text{Hz}}
  \]

Signal/Noise is < 1%
Signal increases with $L$. Large interferometer for gravitational wave detectors have $L \sim 4$ km.

Noise decreases with Power. Even with special glass, loss in beam splitter deforms glass due to thermal heating at $P \sim 3$ kW.

Pushing to this extreme, Signal/Noise $\sim 1$.

We asked LIGO very politely if we could borrow (and reconfigure) an interferometer but they are using it to look for gravitational waves.
Back to Hypothesis: Coherence
Experiment: Holometer Observing Strategy

\[ T_I = \left( \frac{\chi^2}{P_{BS}^2 D^3} \right) \left( \frac{1}{\ell_P^2} \right) \left( \frac{h^2 c^3}{8\pi^3} \right) \sim \text{minutes}. \]

Note: non-gaussian noise in optics and electronics will (probably) dominate exposure time.
Summary of novel ideas

- A new commutator: \( [X_i, X_j] = \dot{\mathbf{i}} \cdot X_k \epsilon_{ijk} \ell' P \)

- Normalization from The Holographic Principle

- \( \ell' P = \ell P / 2 \sqrt{\pi} \)

- Holographic uncertainty adds noise to measurements of the location of the beam splitter in a Michelson interferometer

- \( R \Delta \Delta(\tau) \) for Holographic noise in one interferometer falls from \( R \Delta \Delta(0) = D \ell P / 2 \sqrt{\pi} \) to 0 at \( \tau = \tau_c \)

- Holographic noise of two co-located interferometers is correlated and depends on the overlapping spacetime volume.
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▶ A new commutator: \([X_i, X_j] = iX_k\epsilon_{ijk}\ell'_P\)
▶ Normalization from The Holographic Principle \(\ell'_P = \ell_P/2\sqrt{\pi}\)
▶ Holographic uncertainty adds noise to measurements of the location of the beam splitter in a Michelson interferometer
▶ \(R_{\Delta\Delta}(\tau)\) for Holographic noise in one interferometer falls from \(R_{\Delta\Delta}(0) = D\ell_P/2\sqrt{\pi}\) to 0 at \(\tau = \tau_c\)
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Conclusion

We have designed, assembled and are currently commissioning the Fermilab Holometer. We will be sensitive to the macroscopic effects of these ideas:

- magnitude of the cross correlation;
- spectral shape of the cross correlation;
- modulation of the cross correlation with the configuration of the interferometers.