The New Muon g-2 Experiment at Fermilab

Dave Kawall, University of Massachusetts Amherst

**Goal:** Measure the muon anomalous magnetic moment, $a_\mu$, to 0.14 ppm, a fourfold improvement over the 0.54 ppm precision of Brookhaven E821
**What is the world made of?** Standard Model includes $\approx 200$ particles

- These particles all formed from elementary building blocks

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Leptons</th>
<th>Gauge bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>charge</td>
<td>name</td>
</tr>
<tr>
<td>$2.4 \text{ MeV/c}^2$</td>
<td>$\frac{1}{2}$</td>
<td>up</td>
</tr>
<tr>
<td>$1.27 \text{ GeV/c}^2$</td>
<td>$\frac{1}{2}$</td>
<td>charm</td>
</tr>
<tr>
<td>$171.2 \text{ GeV/c}^2$</td>
<td>$\frac{1}{2}$</td>
<td>top</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>photon</td>
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<td>$1 \text{ GeV/c}^2$</td>
<td>$0$</td>
<td>Higgs boson</td>
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<tr>
<td>$4.8 \text{ MeV/c}^2$</td>
<td>$\frac{1}{2}$</td>
<td>d</td>
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<tr>
<td>$104 \text{ MeV/c}^2$</td>
<td>$-\frac{1}{2}$</td>
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<tr>
<td>$4.2 \text{ GeV/c}^2$</td>
<td>$\frac{1}{2}$</td>
<td>b</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>g</td>
</tr>
<tr>
<td>$1$</td>
<td>$0$</td>
<td>H</td>
</tr>
</tbody>
</table>

- Higgs boson discovered, completes Standard Model : spin $0^+$, March 2013
- All visible matter is made up of the first generation of matter particles
- No idea why there are three generations
- Anti-matter equivalents too
- Periodic table spans range of $\approx 240$ in mass
- Elementary particle masses span sub-eV to $171 \text{ GeV}$, range of $10^{11}$
- Particle masses not understood

- Three generations of spin $\frac{1}{2}$ fermions, four spin 1 gauge bosons

Wikipedia
The Standard Model of Particle Physics: 2013

- Behavior of generations is similar despite large differences in mass
- Explained by symmetries deep in the structure of Standard Model

- Symmetries explain similarity - but not why there are 3 generations

\[ \tau \]

\[ \mu \]

\[ e \]

\[ \mu \rightarrow e + \gamma < 5.7 \times 10^{-13} \] (90% CL)

- Perhaps 2nd, 3rd generations are excited states of 1st?

Invariant under laws of chemistry

Invariant under electroweak interaction
The Standard Model of Particle Physics: 2013

- Standard Model: theory of EM, weak, and strong interactions of fundamental particles
- Makes precise predictions, describes experimental results: incredibly successful

SM predictions of Higgs decay channels

\[ \sigma / \sigma_{SM} = 0.88 \pm 0.21 \]

(A. Whitbeck, QCD Moriond, Mar 14, 2013)
• Standard Model does not describe gravity
• Why 3 generations of fermions? Why more matter than antimatter?
• Hierarchy problems:
  • Why is weak force $10^{30+}$ times stronger than gravity?
  • Why Higgs so light; 126 GeV vs $10^{16}$-$10^{19}$ GeV?
• Planck CMB: 4.9% Ordinary matter, 26.8% Dark Matter, 68.3% Dark Energy
  • What is Dark Matter? What is Dark Energy? We’re only 5% done.
How do we solve these mysteries?

- **Energy Frontier**: Colliders SLC, LEP, Tevatron, DESY, LHC: directly produce particles of Standard Model - but can’t provide all the answers

- **Low energy tests important**: neutrino oscillations, dark matter ...

The KATRIN experiment is designed to measure the mass of the electron neutrino directly with a sensitivity of 0.2 eV. It is a next generation tritium beta-decay experiment scaling up the size and precision of previous experiments by an order of magnitude as well as the intensity of the tritium beta source.

- **Intensity Frontier**: look for new phenomena or ultra-rare processes that shouldn’t happen in SM ($\mu N \rightarrow eN$, proton decay))

- **Fundamental Symmetries**: look for violations of symmetries incompatible with Standard Model: EDMs, Local Lorentz Invariance, new sources CP-violation, ...

- **Precision Frontier**: look for variation of constants, deviations from Standard Model predictions, ...
Magnetic Dipole Moments

• Classically, magnetic dipole moment of current loop given by:

\[
\vec{m} = \frac{IA}{c} \hat{n} = \frac{1}{c} \frac{ev}{2\pi r} \pi r^2 \hat{n} = \frac{e}{2mc} mvr \hat{n}
\]

\[
\vec{m} = \frac{e}{2mc} \vec{L}, \quad \vec{m} = g \frac{e}{2mc} \vec{S},
\]

• Bohr Magneton: magnetic moment of an electron with 1 ħ of angular momentum:

\[
\mu_B = \frac{e\hbar}{2m_e c}, \quad g = 1
\]

• But: spectroscopy indicated anomalous Zeeman transitions (involving electron spin in a magnetic field) \( \Rightarrow \) required \( g=2 \)!

• Explained by relativistic treatment of quantum mechanics by Dirac in 1928:
  \( \Rightarrow \) predicted \( g=2 \) for fundamental spin 1/2 particle, huge success!

• 1933 Otto Stern measured proton \( \mu_p \): required \( g=5.6 \)!

• Later recognized as first indication of proton substructure. \( g \) factors have been important
Anomalous part of the Magnetic Moment

- Recall magnetic moment interaction \( H_{\text{Zeeman}} = -\mu \cdot B \)

\[
\mu = -g \frac{e}{2mc} S, \quad S = \frac{\hbar}{2} \sigma \quad \text{from quantum mechanics}
\]

- Dimensionless \( g \)-factor can be predicted from theory

- 1947: 0.1% discrepancies in spectroscopy. G. Breit suggests \( g_e = 2 + \epsilon \)

- 1948: Measurements of Kusch and Foley found \( g_e \) deviates from 2

- 1948: Schwinger QED calculation of \textit{anomalous part of} \( g_e \) factor, \( a_e \) where \( g_e \equiv 2(1 + a_e) \)

- \( a_e = \alpha/2\pi \approx 0.00116 \) due to \textit{radiative corrections} from virtual particles in loops

- 1 part in 850 effect, huge success for QED!
New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

D. Hanneke, S. Fogwell, and G. Gabrielse*
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
(Received 4 January 2008; published 26 March 2008)

A measurement using a one-electron quantum cyclotron gives the electron magnetic moment in Bohr magnetons, \( g/2 = 1.001159\,652\,180\,73(28)\) [0.28 ppt], with an uncertainty 2.7 and 15 times smaller than for previous measurements in 2006 and 1987. The electron is used as a magnetometer to allow line shape statistics to accumulate, and its spontaneous emission rate determines the correction for its interaction with a cylindrical trap cavity. The new measurement and QED theory determine the fine structure constant, with \( \alpha^{-1} = 137.035\,999\,084(51)\) [0.37 ppb], and an uncertainty 20 times smaller than for any independent determination of \( \alpha \).

- \( g_e \) most precisely known quantity in physics, to 0.28 ppt

- Penning trap for single electron
- Magnetic confinement in radial
- Electrodes for vertical confinement
- Trapped for months
- \( a_e = (g_e - 2)/2 \) determined to 0.24 ppb

FIG. 2 (color). Cylindrical Penning trap cavity used to confine a single electron and inhibit spontaneous emission.
Theory of the Anomalous Magnetic Moment of the Electron

\[
\frac{g_e}{2} = 1 + C_2 \left( \frac{\alpha}{\pi} \right) + C_4 \left( \frac{\alpha}{\pi} \right)^2 + \ldots + C_{10} \left( \frac{\alpha}{\pi} \right)^5 + \ldots + a_{\mu,\tau} + a_{\text{hadonic}} + a_{\text{weak}}
\]


- Extract \( \alpha \), compare with other measurements, confirms QED at ppt level

\[ a_e = \frac{(g_e - 2)}{2} \text{ determined to 0.24 ppb} \]

- Muons live 2.2 \( \mu \)seconds - why bother measuring \( a_\mu \)?

- Sensitivity to new physics: \( \Delta a_{e,\mu}(\text{New Physics}) \approx C \left( \frac{m_{e,\mu}}{\Lambda} \right)^2 \)

\( \Rightarrow \) Muon mass 206 times electron mass, so new physics contribution 40,000 times larger

\( \Rightarrow \) New physics contribution of 0.24 ppb on \( a_e \) corresponds roughly to 9 ppm on \( a_\mu \)

- \( a_\mu \) known from Brookhaven E821 to 0.54 ppm, hope to push at Fermilab to 0.14 ppm
Contributions to the Anomalous Magnetic Moment of the Muon

\[ a_\mu (\text{Standard Model}) = a_\mu (\text{QED}) + a_\mu (\text{Weak}) + a_\mu (\text{Hadronic}) \]

EW 1 Loop

EW 2 Loop

Hadronic Leading Order

Higher Order

Light-by-Light

⇒ \( a_\mu \) gets contributions from all physics - including the unknown
\( a_{\mu}^{\text{had;LO}} \) can be extracted from measurements by SND, CMD2, BaBar, KLOE, Belle

\[
a_{\mu}^{\text{had;LO}} = \left( \frac{\alpha m_{\mu}}{3\pi} \right)^2 \int_{4m_{\pi}^2}^{\infty} \frac{ds}{s^2} K(s) R(s), \quad \text{where} \quad R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}
\]

- CMD3 will measure up to 2.0 GeV, using energy scan and ISR, good cross-check
- KLOE will measure \( \gamma^*\gamma^* \rightarrow \pi^0 \), might reduce uncertainty on \( a_{\mu}(\text{Had;LBL}) \)


\[
\begin{align*}
a_{\mu}(\text{QED}) & = 116\ 584\ 718.951 \pm 0.080(\alpha^5) \\
a_{\mu}(\text{HadVP; LO}) & = 6\ 923. \quad \pm 42(\text{Exp}) \\
a_{\mu}(\text{HadVP; LO}) & = 6\ 949. \quad \pm 43(\text{Exp}) \\
a_{\mu}(\text{HadVP; HO}) & = -98.4 \quad \pm 0.6(\text{Exp}) \pm 0.4(\text{Rad}) \\
a_{\mu}(\text{Had; LBL}) & = 105. \quad \pm 26 \\
a_{\mu}(\text{Weak; 1 loop}) & = 194.8 \\
a_{\mu}(\text{Weak; 2 loop}) & = -41.2 \quad \pm 1(\text{Had}) \pm 2 \rightarrow 0(\text{Higgs})
\end{align*}
\]

\[\Rightarrow a_{\mu}(\text{SM}) = 116\ 591\ 802. \quad \pm 49 \times 10^{-11} \quad (0.42 \text{ ppm})\]

\[\Rightarrow a_{\mu}(\text{SM}) = 116\ 591\ 828. \quad \pm 50 \times 10^{-11} \quad (0.43 \text{ ppm})\]
In units of $10^{-11}$:

\[
\begin{align*}
    a_\mu(\text{Expt}) &= 116\,592\,089 \pm 54 \pm 33 \text{ (0.54 ppm)} \\
    a_\mu(\text{SM}) &= 116\,591\,802 \pm 49 \text{ (0.42 ppm)} \\
    a_\mu(\text{SM}) &= 116\,591\,828 \pm 50 \text{ (0.43 ppm)}
\end{align*}
\]

\[
\begin{align*}
    a_\mu(\text{Expt}) - a_\mu(\text{SM}) &= 287\pm 80 \text{ (3.6}\sigma) \\
    a_\mu(\text{Expt}) - a_\mu(\text{SM}) &= 261\pm 80 \text{ (3.3}\sigma)
\end{align*}
\]

⇒ Deviation is large compared to weak contribution and uncertainty on hadronic terms
⇒ Signature of new physics?
⇒ Deviation doesn’t reach 5$\sigma$ threshold for discovery - need to reduce uncertainties
⇒ Need to do a better experiment! Need to reduce theoretical uncertainties
Low Energy Precision Tests: Beyond the Standard Model

- $a_\mu$ is sensitive to variety of new physics; including many SUSY models

\[
\Delta a_\mu(\text{SUSY}) \simeq (\text{sgn}\mu) \times (130 \times 10^{-11}) \times \tan \beta \times \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2
\]

⇒ $\mu$ and $\tan \beta$ are difficult to measure at LHC, $g_\mu - 2$ can provide tighter constraints

- Snowmass Points and Slopes take benchmark points in SUSY parameter space and predict observables
- Muon $g-2$ is a powerful discriminator amongst models of physics
- Regardless of the final value, it strongly constrains all the possibilities
• Many well motivated theories predict large $\Delta a_\mu$ - new g-2 can constrain parameters
• Many well motivated theories predict tiny $\Delta a_\mu$ - if large $\Delta a_\mu$ found by new g-2, these are excluded
• Some models predict similar signatures at LHC but distinguishable by $\Delta a_\mu$ (MSSM and UED (1D), Littlest Higgs)
• New g-2 sensitive to parameters difficult to measure at LHC [ $\tan(\beta)$, $\text{sgn}(\mu)$ ]
• Provides constraints on new physics that are independent and complementary to LHC, CLFV ($\mu \rightarrow e$), EDMs, ...

⇒ Even agreement with the Standard Model would be very interesting
⇒ Sensitivity to new particles with TeV scale mass

⇒ Many reasons to pursue a new measurement of $a_\mu$ at Fermilab, reduce $\delta a_\mu$ from 0.54 ppm $\rightarrow$ 0.14 ppm
• E989 will measure the Muon Anomalous Magnetic Moment to $\pm 0.14$ ppm precision
• Factor of 4 improvement possible due to advantages at Fermilab
Overview of the the Experimental Method

• Produce an 8 GeV pulsed proton beam, direct it onto a production target
• Capture pions from production target
• Capture muons >90% polarized from “forward” pion decay $\pi^+ \rightarrow \mu^+ \nu_\mu$
• Transport muon beam, inject it into g-2 storage ring
• Kick the muon beam onto a stored orbit
• Measure arrival time and energy of positron from muon decay in ring $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$

• Determine $a_\mu = (g_\mu - 2)/2$ from the spin equation:

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \left( \frac{mc}{p} \right)^2 \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

• Spin vector precesses differently from momentum vector precessing at cyclotron frequency
• Difference directly sensitive to $a_\mu \approx \alpha/2\pi \approx 0.00116...$, not $g_\mu \approx 2.00232...$
• Cancel term from electrostatic vertical focusing at $p_{\text{magic}} = m_\mu c / \sqrt{a_\mu} \approx 3.094 \text{ GeV}/c$

$\Rightarrow$ Need to measure muon spin frequency $\omega_a$ and magnetic field $\vec{B}$ averaged over muon distribution in ring
The g-2 Experiment in a Nutshell

Overview of the g-2 experiment

- 8 GeV Protons From Booster
- Hit Target.
- Muons enter storage ring through a SC inflector that cancels storage ring $B$ field.
- Muons kicked onto orbit by pulsed magnetic field.
- Muons confined vertically by electric quadrupoles.
- Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.
- After each circle, muon's spin axis changes by 12°, yet it keeps on traveling in the same direction.
- Muons are tiny magnets spinning on axis like tops.
- Pions, weighing 1/6 proton, are created.
- Pions decay to muons.
- One of 24 detectors see an electron, giving the muon spin direction; $g$-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.
- After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.

- Just like $a_e$ use a Penning trap, except 7.112 meter radius, 650 tons.
- Muons enter storage ring through a SC inflector that cancels storage ring $B$ field.
- Muons kicked onto orbit by pulsed magnetic field.
- Muons confined vertically by electric quadrupoles.
Experimental Procedure: Based on BNL E821 Muon $g_\mu - 2$ Experiment

- Inject polarized muons at 3.094 GeV/c into superferric storage ring, radius = 711.2 cm
- Muon spin precesses in homogeneous 1.45 T field, time dilated lifetime of 64.4 $\mu$s, measure for 700 $\mu$s

$$\bar{\omega}_a = \bar{\omega}_s - \bar{\omega}_c : \text{difference between spin and cyclotron frequencies}$$

$$\bar{\omega}_a = -\frac{q}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \quad \Rightarrow \quad \text{at } \gamma = 29.3 \quad \Rightarrow \quad \bar{\omega}_a = -\frac{q}{mc} \left[ a_\mu \vec{B} \right]$$

$\Rightarrow$ To determine $a_\mu$, need to measure $\omega_a$ and $B$ (weighted by muon distribution)

- Sub-ppm corrections applied due to vertical betatron motion (pitch correction) and muons not at magic $\gamma$
Superconducting Inflector

- Have to get the muon beam into the storage ring - from low/fringe field area outside to 1.45 T inside - beam strongly deflected unless we cancel this field
- Use a superconducting flux-exclusion tube? Perturbations in storage region too large
Superconducting Inflector

• Base plan: use double-$\cos \theta$ design from BNL E821, $\int \vec{B} \cdot d\vec{L} = 2.55 \, \text{T} \cdot \text{m}$

![Diagram of Superconducting Inflector](image)

• Procedure:
  • Warm inflector + Type II SC shield, turn on main magnet, flux penetrates inflector and SC shield
  • Cool inflector and shield, since $H > H_{C1}$ field fully penetrates shield
  • Energize coils - cancels field in beam channel, eddy currents in passive shield prevents flux leaking out
  • Cancels $B$ field in beam channel, no perturbation to field outside SC shield
Superconducting Inflector

- Versions of superconducting inflector with closed and open ends

- BNL E821 inflector closed ends, significant multiple scattering, aperture $18 \times 56 \text{ mm}^2$, injection efficiency $\approx 2\%$

- New inflector: open ends, $40 \times 56 \text{ mm}^2$ (storage aperture $\pm 45\text{ mm}$) $\rightarrow$ $4 \times$ more stored muons, could do $\mu^-$ and $\mu^+$
The Fast Muon Kicker

- Muons exit the inflector, enter storage region at radius 77 mm outside ideal closed orbit
- Muons cross ideal orbit $\approx 90^\circ$ later in azimuth, angle off by 10.8 mrad
- Including momentum spread, multiple scattering in inflector, need 14 mrad kick
- Temporarily reduce $B$ by 280 Gauss, $\int B \cdot dL = 1.4 \text{ kG}\cdot\text{m}$ for 14 mrad kick
- Pulse width $80 \text{ ns} < \tau < 149 \text{ ns}$, 100 Hz, 10% homogeneity
The Fast Muon Kicker

- New geometry yields 33%-50% higher field/current than BNL E821
- $3 \times 1.7$ m stripline kickers, Blumlein PFN
- Tracking studies determine optimal shape
- Dave Rubin and collaborators at Cornell
Storage ring is a weak-focusing betatron using electric quadrupoles for linear restoring force in vertical, \( \kappa = \frac{dE_y}{dy} \), Field Index \( n = \frac{\kappa R_0}{\beta B_0} \approx 0.137 \).

Uniform quadrupole field leads to simple harmonic motion - radial \( x \) and vertical \( y \) betatron oscillations of beam.

\[
x = x_e + A_x \cos \left( \nu_x \frac{s}{R_0} + \delta_x \right), \quad y = A_y \cos \left( \nu_y \frac{s}{R_0} + \delta_y \right)
\]
Stored Beam Dynamics and Related Systematic Uncertainties

\[ x = x_c + A_x \cos \left( \nu_x \frac{s}{R_0} + \delta_x \right), \quad y = A_y \cos \left( \nu_y \frac{s}{R_0} + \delta_y \right) \]

\[ \nu_x = \sqrt{1-n}, \quad \nu_y = \sqrt{n}, \quad n \approx 0.137, \quad f_x = f_C \sqrt{(1-n)} \approx 0.929 f_C, \quad f_y = f_C \sqrt{n} \approx 0.37 f_C \]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expression</th>
<th>Frequency [MHz]</th>
<th>Period [\mu s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_a )</td>
<td>( \frac{e}{2\pi mc} a_{\mu} B )</td>
<td>0.228</td>
<td>4.37</td>
</tr>
<tr>
<td>( f_C )</td>
<td>( \frac{v}{\pi R_0} )</td>
<td>6.7</td>
<td>0.149</td>
</tr>
<tr>
<td>( f_x )</td>
<td>( \sqrt{1-n} f_c )</td>
<td>6.23</td>
<td>0.160</td>
</tr>
<tr>
<td>( f_y )</td>
<td>( \sqrt{n} f_c )</td>
<td>2.48</td>
<td>0.402</td>
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<tr>
<td>( f_{CBO} )</td>
<td>( f_c - f_x )</td>
<td>0.477</td>
<td>2.10</td>
</tr>
<tr>
<td>( f_{VW} )</td>
<td>( f_c - 2f_y )</td>
<td>1.74</td>
<td>0.574</td>
</tr>
</tbody>
</table>

- Perturbations of stored muon beam from ideal circular orbit affect \( \omega_a \)

\[ \Rightarrow \text{Resonances in ring cause muon beam losses, distort time spectrum} \]

- Resonances occur if \( L \nu_x + M \nu_y = N \) where \( L, M, N \) integers. Operating points have \( \nu_x^2 + \nu_y^2 = 1 \)
How will we measure $\omega_a$?

- To measure $\omega_a$, need to know muon spin direction when it decayed
- Nature is kind here: muon decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ is self-analyzing due to PV
- Muon spin direction correlated with decay positron direction

Averaged over all positron energies, forward-backward asymmetry wrt muon spin is $a=1/3$

For highest energy positrons (3.1 GeV), asymmetry $a=1$

$E_{\text{lab}} \approx \gamma E^* (1 + \cos \theta^*)$

Detect decay $e^+$ above 1.8 GeV $\Leftrightarrow$ cut on $\theta^*$, reconstruct muon spin direction versus time

Figures from thesis of Alex Grossmann
• Need fast calorimeter to detect $e^+$ from muon decay - made from PbF$_2$ crystals
• Each calorimeter segmented into $9 \times 6$ individual crystals to handle pileup
• Čerenkov light detection with silicon photomultipliers (SiPMs)
• Smaller Moliere radius, greater segmentation, greater immunity to pileup then BNL E821
• Signals digitized with 500 MHz waveform digitizers for 700+µs, extract $e^+$ signals offline
Measurement of $\omega_a$

\[ N_{\text{ideal}}(t) = N_0 \exp\left(-t/\gamma\tau_\mu\right) \left[1 - A \cos(\omega_a t + \phi)\right] \]

\[ \frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a \tau_\mu A \sqrt{N}} \]

- $3.6 \times 10^9 e^+$, corrections for muon losses, pileup, coherent betatron oscillations
Coherent Betatron Oscillations (CBO)

- Detector acceptance depends on muon radius at decay - coherent radial motion modulates electron time spectrum
- Radial betatron wavelength (blue line) is longer than circumference (cyclotron wavelength), $f_x < f_C$
- At fixed detector location, each pass of bunched beam appears at different radius - moving at $f_{CBO}$
- CBO frequency $f_{CBO} = f_C - f_x$ must be kept far from $f_a$

CBO frequency $f_{CBO} = f_C - f_x$ must be kept far from $f_a$

- Cyclotron wavelength marked by black lines, single detector by black block, betatron oscillations in blue
- Red line: apparent radial breathing in and out of beam at $f_{CBO}$
- Effect nearly cancels when all detectors added together
Coherent Betatron Oscillations (CBO)

- In BNL E821 2000 data taken when CBO frequency close to $f_\alpha$ - can be seen in residual to 5 parameter fit
- In 2001, field index $n$ changed to move $f_{CBO}$ away from $f_\alpha$
Corrections to $\omega_a$: Radial Electric Field Correction

$$\tilde{\omega}_a \approx \tilde{\omega}_S - \tilde{\omega}_C = -\frac{e}{m} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

- Not all muons at magic momentum, $p = p_m + \Delta p$,
- Storage ring momentum acceptance $\Delta p \approx \pm 0.5\%\ p_m$, $p_m \approx 3.094\ \text{GeV}/c$

$$\frac{p - p_m}{p_m} = (1 - n) \left[ \frac{R - R_0}{R_0} \right] = (1 - n) \frac{x_e}{R_0}$$

$$\frac{\omega_a' - \omega_a}{\omega_a} = \frac{\Delta \omega_a}{\omega_a} = -2 \frac{\beta E_r}{c B_y} \left( \frac{\Delta p}{p_m} \right) = -2n(1 - n) \beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

- Momentum distribution from fast-rotation (de-bunching) analysis, decay $e^+$ tracking chambers, muon beam fiber monitors
Corrections to $\omega_a$: Radial Electric Field Correction

- Momentum distribution from fast-rotation (de-bunching) analysis, decay $e^+$ tracking chambers, muon beam fiber monitors

- Bunch structure visible at early times, $\tau_{\text{cyclotron}} \approx 149$ ns
- Bunch structure erased by $60 \mu s$ due to momentum spread $\Delta p$
- BNL E821 injected beam width $\approx 23$ ns
- BNL E821 $\sqrt{\langle x_e^2 \rangle} \approx 10$ mm $\Rightarrow$ electric field correction $+0.47 \pm 0.05$ ppm
- FNAL E989 beam width $\gg 23$ ns
- FNAL E989 uncertainty on correction $\pm 0.03$ ppm using improved traceback system
Corrections to $\omega_a$: Pitch Correction

\[ \tilde{\omega}_a \approx \tilde{\omega}_S - \tilde{\omega}_C = -\frac{e}{m} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] \]

- Vertical betatron motion
  - $\vec{\beta}$ not perpendicular to $\vec{B}$

\[ \omega'_a \approx \omega_a \left( 1 - \frac{\psi^2}{2} \right) \]

\[ C_p = -\frac{<\psi^2>}{2} = -\frac{n <y^2>}{2 \cdot R_0^2} \]

- Electric field and pitch corrections reduce observed frequency
- BNL E821 pitch correction $+0.27 \pm 0.036$ ppm
- Electric field and pitch are the only corrections made to the $\omega_a$ data
- Improved E989 muon tracking reduces uncertainties $\pm 0.05$ ppm $\Rightarrow 0.03$ ppm level
Projected Systematic Uncertainties on $\omega_a$ and Comparison with E821

- The largest systematics uncertainties on $\omega_a$ from the final E821 run, and projected future uncertainties are outlined:

<table>
<thead>
<tr>
<th>E821 Error</th>
<th>Size [ppm]</th>
<th>Plan for the New $g-2$ Experiment</th>
<th>Goal [ppm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain changes</td>
<td>0.12</td>
<td>Better laser calibration and low-energy threshold</td>
<td>0.02</td>
</tr>
<tr>
<td>Lost muons</td>
<td>0.09</td>
<td>Long beamline eliminates non-standard muons</td>
<td>0.02</td>
</tr>
<tr>
<td>Pileup</td>
<td>0.08</td>
<td>Low-energy samples recorded; calorimeter segmentation</td>
<td>0.04</td>
</tr>
<tr>
<td>CBO</td>
<td>0.07</td>
<td>New scraping scheme; damping scheme implemented</td>
<td>0.04</td>
</tr>
<tr>
<td>$E$ and pitch</td>
<td>0.05</td>
<td>Improved measurement with traceback</td>
<td>0.03</td>
</tr>
<tr>
<td>Total</td>
<td>0.18</td>
<td>Quadrature sum</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Introduction to $B$ Field Measurement and $\omega_p$ Systematics

- E989 relies on precision measurement of two quantities, $\omega_a$ and $\tilde{\omega}_p$:

\[ a_\mu = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p} \]

- $\tilde{\omega}_p$ : free proton precession frequency weighted by muon distribution $\approx 2\pi \times 61.79$ MHz

⇒ Goal is to determine $\tilde{\omega}_p$ with uncertainty 0.070 ppm ($\delta\tilde{\omega}_p \leq 2\pi \times 4.3$)

- E989 largely based on principles and hardware developed by Heidelberg and Yale for E821
- E821 fractional uncertainty on field was 0.17 ppm : E989 needs to do 2.4 times better
- Changes to hardware and techniques to get from 0.17 to 0.070 ppm on $\omega_p$ outlined below

### Four Field Measurement Tasks:

1. Monitor magnetic field with fixed probes on vacuum chambers while muons stored in ring;
2. Map the magnetic field in muon storage volume with NMR trolley when the beam is off;
3. Provide an absolute calibration relating NMR trolley field measurements inside storage volume to the precession frequency of a free proton;
4. Provide feedback to the power supply to stabilize field when muon data are collected.
Field Measurement with Pulsed NMR

- Measure field using pulsed NMR to induce and detect free induction decay (FID) of protons in a water sample \(^1\)
- Typical NMR probe shown below (field direction vertical, perpendicular to \(L_s\) coil axis):

\[
\text{RF pulse at } f_{\text{ref}} = 61.74 \text{ MHz produces RF magnetic field in coil } L_s \text{ around sample}
\]
- Rotates magnetization of protons in sample perpendicular to main field
- After pulse, proton spins process freely, coherently at \(f_{\text{NMR}} \approx 61.79 \text{ MHz, } \omega \approx \gamma_p B\)
- Rotating magnetization induces \(V\) in coil \(L_s\), signal decays exponentially, \(\tau \approx 1 \text{ ms}\)

\(^1\)May use petroleum jelly (CAS 8009-03-08) : long \(T_2 \approx 40 \text{ ms, doesn’t evaporate, low temp. coefficient}\)
Field Measurement with Pulsed NMR

- NMR signal at $f_{NMR}$ goes to low noise amplifier, mixed with $f_{ref} = 61.74$ MHz from synthesizer
- Difference frequency $f_{NMR} - f_{ref} \equiv f_{FID}$ ranges from 45-55 kHz, dependent on local field
- Difference of 62 Hz in $f_{FID}$ corresponds to 1 ppm difference in field
- Count zero crossings of this free induction decay (FID) and ticks of clock running at 20 MHz till signal decays to roughly $1/e$ of peak, $\approx 1$ ms

$\Rightarrow$ Local field characterized by Larmor frequency, $f_{NMR} = f_{ref} + f_{FID}$
- Single shot resolution on $f_{NMR}$ $\approx 0.020$ ppm
- Depends on signal duration, $S/N$
Field Measurement with Pulsed NMR

- FID from E821 and Fourier transform

- Signals typically last 1 ms
- Signal : noise \( \geq 100 : 1 \)
- Frequency resolution \( \approx \) linwidth/[S/N]
  \( \approx \frac{130 \text{ Hz}}{100} = 1.3 \text{ Hz} \)
- Resolution on \( f_{\text{NMR}} \) of \( \delta f_{\text{NMR}} = 1.3 \text{ Hz} \)
  \( \delta f_{\text{NMR}}/f_{\text{NMR}} = 1.3 \text{ Hz}/61.79 \text{ MHz} \)
  0.020 ppm resolution on field

- Corrections necessary to get from \( f_{\text{NMR}} \) of protons in \( \text{H}_2\text{O} \) to \( \omega_p \) of free proton

- Need absolute calibration of probes in terms of free proton precession frequency; demonstrated at level of 0.034 ppm (see X. Fei et al., Nucl. Inst. Meth. A 394, 349 (1997))

- Main challenge of field measurement: effectively transfer high accuracy absolute calibration to many probes providing high resolution monitoring field over long periods in which muons are stored
Field Measurement Hardware

- Block diagram of the proposed NMR electronics shown.

- Multiplexer connects to 20 NMR probes, and contains a duplexer and preamplifier

- DL611 frequency counter, NIM modules, multiplexers, NMR probes from E821 will be refurbished for E989; parts shaded red are new
Need Larmor frequency of free protons in storage volume while muons are stored

(1) **Fixed probes** measure field at the same time as muons stored, but outside storage volume

(2) Field inside storage volume measured by **NMR trolley**, but not when muons stored

- Fixed probes are cross-calibrated when trolley goes by; can infer field inside storage volume when muons stored from fixed probes

(3) Trolley probes calibrated in terms of free proton frequency by an absolute calibration probe
Field Measurement Task (3) : Absolute Calibration

- Construct absolute calibration probe with spherical water sample at known temperature
  \[ \omega_p(\text{sph} - \text{H}_2\text{O}, T) = [1 - \sigma(\text{H}_2\text{O}, T)] \omega_p(\text{free}), \]
  \( \sigma(\text{H}_2\text{O}, T) \approx 26 \text{ ppm, is the temperature-dependent diamagnetic shielding of the proton in a water molecule} \)
  \( \bullet \) E821 absolute calibration probe properties known well enough to determine fields in terms of free protons to accuracy of 0.034 ppm

- E821 used this probe with accuracy of 0.050 ppm (limited in part by temp. uncertainties)
- E989 will repeat and improve study of probe properties, *improve temperature stability and monitoring* to reduce temperature related uncertainties, calibration goal is 0.035 ppm
Error budget for the $\omega_p$ measurement

- Systematic errors on E821 field measurements from 1999, 2000, 2001 listed below
- The final column lists the uncertainties anticipated for E989

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute calibration of standard probe</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.035</td>
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<tr>
<td>Calibration of trolley probes</td>
<td>0.20</td>
<td>0.15</td>
<td>0.09</td>
<td>0.03</td>
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<tr>
<td>Trolley measurements of $B_0$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.05</td>
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<td>Interpolation with fixed probes</td>
<td>0.15</td>
<td>0.10</td>
<td>0.07</td>
<td>0.03</td>
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<tr>
<td>Uncertainty from muon distribution</td>
<td>0.12</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
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<tr>
<td>Inflector fringe field uncertainty</td>
<td>0.20</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Time dependent external $B$ fields</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.005</td>
</tr>
<tr>
<td>Others †</td>
<td>0.15</td>
<td>0.10</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>Total systematic error on $\omega_p$</td>
<td>0.4</td>
<td>0.24</td>
<td>0.17</td>
<td>0.070</td>
</tr>
<tr>
<td>Muon-averaged field [Hz]: $\bar{\omega}_p/2\pi$</td>
<td>61 791 256</td>
<td>61 791 595</td>
<td>61 791 400</td>
<td>–</td>
</tr>
</tbody>
</table>

- †Higher multipoles, trolley temperature ($\leq 0.05$ ppm/°C) and power supply voltage response (0.4 ppm/V, $\Delta V=50$ mV), and eddy currents from the kicker.
- Note the steady reduction in uncertainties achieved in E821
The Future: E989 at the new Muon Campus at Fermilab

Future g-2 Experimental Hall

Future Mu2e Experimental Hall

Wilson Hall
E989 : Fermilab offers advantages, factor 4 improvement possible

Recycler
- Rebunches 8 GeV protons from booster

Target Station
- Target + focusing lens

Decay Line
- Target to M2 to M3 to delivery ring
- 900 m long decay channel for $\pi \rightarrow \mu$
  - reduced $\pi$ and $p$ in ring,
  - factor 20 reduction in hadronic flash
- $4 \times$ higher fill frequency than E821
- Muons per fill about the same
- 21 times more detected $e^+, \ 2 \times 10^{11}$
- Better temperature control in experimental hall
- Reduction in systematics by factor of 3 without major modifications
- 650 ton magnet iron yoke and pole pieces are being disassembled, transported by barge
- 8 ton, 15 m diameter superconducting coils must be transported in one piece
Transporting the coils to FNAL:

- Trailer with coils passes toll arches with 6” clearance on each side
- “Nature is hard and unyielding” - Martin Perl, *Reflections on Experimental Science*
- We were lucky this time
Extensive review process. Total project cost is \( \approx $40 \) million: nearly 1/2 is for upgrade of accelerator and new multipurpose facilities at FNAL.

First data in 2016? Run for 2 or more years.

Precision measurements take a lot of patience - but they’re worth it.
Summary

- Experiment under development to measure $a_\mu$ to 0.14 ppm, fourfold improvement over BNL E821
- Reduction in statistical uncertainty by factor 4; reduce $\omega_a$, $\omega_p$ systematics by factor 3
- Hope to motivate improvements in theory and more exp. work:
  - Currently $\delta a_\mu$(HadVP,LO) = 0.36 ppm, and $\delta a_\mu$(Had,LBL) = 0.23 ppm
- Before E821 (1983), expt. known to 7 ppm, theory to 9 ppm: now 0.54 and 0.42 ppm
- Regardless of where final result for $a_\mu$ lands:
  - Precision test Standard Model
  - Determine parameters ($\tan(\beta)$) or viability of many new physics models predicting $\Delta a_\mu \neq 0$ (SUSY models)
  - UED (1D) predict tiny effects incompatible with $\Delta a_\mu << 300 \times 10^{-11}$
  - Constraint on all future models
  - Provide complementary information to direct searches at LHC, CLFV, EDMs